This equation corresponds to the general form of the burden objective function defined by eqn. (4). Similarly, if the analysis were carried out at the Impact Assessment level, the burden objective function would be replaced by the impact objective, as defined by eqn. (5).

System optimisation

As mentioned above, LP optimisation can be used to identify both economic and environmental optima of the system. If the focus of the analysis is the former, then the system in this example is optimised on the profit objective defined by (A.8), subject to constraints (A.1)-(A.7), to give the solution shown in **Table A.1**. Total profit of £50000 is achieved by using feeds x_1 and x_4 only, while the other feeds are not being used in the system. The active constraints for this operating state of the system are defined by eqns. (A.3) and (A.5)-(A.6). The constraint (A.4) is non-active and has a slack value, i.e. unused amount of feed, of 700 kg.

The total burden is also calculated at the solution of the model and is equal to 35 kg. This is equivalent to a conventional way of calculating the burdens in the Inventory stage, as most economic systems are operated around the point of optimum profit. Moreover, the marginal values are also calculated with the total burden to give the marginal allocation coefficients, as discussed in the main text. As only active constraints have non-zero marginal values, this means that the burdens are allocated among the Product 1, Product 2 and the Feed category 1 availability constraints. Their respective values (not shown in *Table A.1*) are 0.0325, 0.0325 and -0.02. This means that an increase in the production of either of the two products by one kg would increase the total burden by 0.0325 kg. However, as the marginal value of the Feed category 1 availability constraint has negative value, an increase of 1 kg in the feed availability would cause a 0.02 kg decrease of the burden. Thus, this simple example demonstrates that the burden does not have to be allocated fully to the products, but to the other constraints as well, depending on what determines the system operation.

Table A.1: Optimisation results for the LP example

Activity	Optimised on Profit	Optimised on Burden
Profit, F (£)	50000	26000
Burden, B (kg)	35	25
Feed 1, x_1 (kg)	1500	800
Feed 2, x_2 (kg)	0	0
Feed 3, x_3 (kg)	0	1200
Feed 4, x_4 (kg)	500	0
Feed 5, x_5 (kg)	0	0
Product 1, x_6 (kg)	1000	1000
Product 2, x_7 (kg)	1000	1000
Heat, Q (MJ)	200	320

If the goal of the study is to identify options for environmental improvements as part of Improvement Assessment, the system is optimised on the burden (or impact) objective function. The results of this optimisation are also shown in **Table A.1**. For the same output of the products, the system operation is now determined by the active constraints (A.5) and (A.6). The total burden is equal to 25 kg, which represents an improvement of 29% over the burden obtained in the optimisation on the profit. This reduction is achieved by using 700 kg less of the feed x_1 compared to the previous case and using x_3 instead of x_4. However, the profit is reduced by about 48%. In addition, the total energy used in the system increases from 200 MJ to 320 MJ. Therefore, in this particular case, the reduction in one burden leads to an increase in another. To help resolve these conflicting situation, the system can be optimised on a number of environmental and socio-economic objectives function. The resulting Pareto or non-inferior surface can then be used to trade-off the environmental and economic performance between the points of minimum burden and maximum profit to identify the BPEO not entailing excessive costs.

Erratum

Int. J. LCA (5) 266 – 272 (1998) "Einstein's Lessons for Energy Accounting in LCA"
by Rolf Frischknecht, Reinout Heijungs and Patrick Hofstetter
on page 286 in the headline of section 4: The correct text is "$E = mc^2$"